Performance Analysis of DTMF Detection using Modified Goertzel Algorithm over AWGN

H.A. Eldenferia, J.R. Elbergali

Electronic Engineering Department- College of Industrial Technology

Misurat- Libya

hamza.ali1985@yahoo.com & jelbergali@yahoo.com

Abstract: Dual-tone multi-frequency (DTMF) signal has been broadly used in the recent communication systems like touch-tone telephones as well as many other applications such as interactive control, telephone banking, and email application. This paper analyzes generation and detection of dual-tone multi frequency using modified Geortzel algorithm and its immunity towards noise. Noise immunity is considered by adding random noise to DTMF signal and detects the Key Error Rate (KER). This paper also provides a MATLAB simulation of modified DTMF detection with added noise and the performance results has been analyzed using Signal-to-Noise-Ratio (SNR) versus Key Error Rate (KER).

***Index Terms*—Goertzel algorithm, DTMF, Signal-to-Noise-Ratio (SNR), Key Error Rate (KER)., MATLAB simulation.**

I. Introduction:

DUAL Tone Multiple Frequency (DTMF) signaling is used in telephone calling, digital answer machines, banking systems [1] [2]. DTMF signaling represents each symbol on a telephone touch-tone keypad (0-9,*, #) as a combination of two sinusoidal tones, as shown in Figure 1. When a key is pressed, a DTMF signal consisting of a row frequency tone plus a column frequency tone is transmitted. The purpose of DTMF decoding is to detect DTMF encoded sinusoidal signals in the presence of noise. The detector offered in this paper implements a methodology based on the well-known Goertzel algorithm, providing an effective way to implement a DTMF detector and decoder. This paper studies the effect of random noise on the key detection and its relationship with Signal-to-Noise-Ratio (SNR).

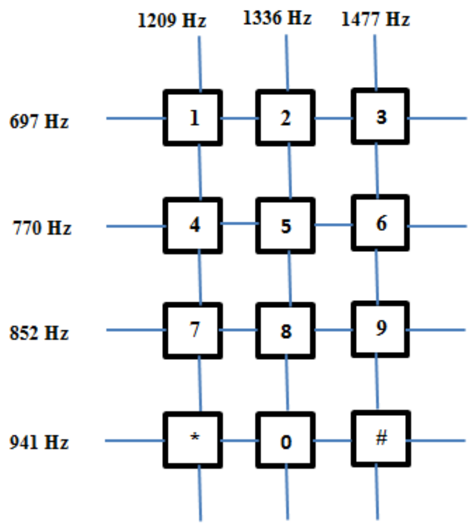

Fig 1. DTMF tone specifications.

II. DTMF Tones Generation and Detection:

DTMF tone contains two superimposed sinusoidal waveforms summed together from a set of standardized frequencies [3]. These frequencies consist of two frequency groups: low frequency group and high frequency group. The keypad of a push-button telephone set is arranged into a 3 × 4 matrix that selects the appropriate pair of frequencies to be transmitted; four low frequency tones (< 1 kHz) are assigned to rows, while three high frequency tones (> 1 kHz) [2] are assigned to columns [1] [3]. This allows a touch tone keypad to have up to twelve unique DTMF tones. Upon pushing a button, two frequencies are transmitted corresponding to the column-row intercept at that button. The general formula for a pure DTMF signal is characterized by the following equation:

$$x(t) = A_M \sin(2\pi f_L t) + A_M \sin(2\pi f_H t) \quad (1)$$

Where A_M is the amplitude assumed to be equal for each DTMF signal, f_L and f_H are the low and high frequencies. The transmitter of a DTMF signal simultaneously sends one frequency from the high-group and one frequency from the low-group. This pair of signals represents the digit or symbol shown at the intersection of row and column in Figure 1. For example, sending 1209Hz and 770Hz indicates that the "4" digit is being sent.

This paper examines "DTMF detection by modified Goertzel algorithm. The first touch tone telephone installation was in 1963. DTMF signaling uses voice-band tones to send address signals and other digital information from pushbutton telephones. Analog DTMF detection is done using band-pass filter banks with center frequencies at the DTMF signal frequencies [4]. Digital detection of DTMF is done by several algorithms like Goertzel, notch filter etc.

III. Goertzel Algorithm

The Goertzel algorithm has been used to design the DTMF tone detector. This powerful algorithm used for calculating Discrete Fourier Transform (DFT) coefficients using a digital filtering method without involving complex algebra like the DFT algorithm. The second-order IIR digital Goertzel filter has been used to build Goertzel algorithm [5], the transfer function is given by [1]:

$$H_k(z) = \frac{Y_k(z)}{X(z)} = \frac{1 - W_N^k z^{-1}}{1 - 2\cos\left(\frac{2\pi k}{N}\right) z^{-1} + z^{-2}} \quad (2)$$

Where N the total number of samples, $x(n)$ is the input data, where n= 0,1…..N-1, and the last element set to be $x(N) = 0$.

Notice that $W_N^k = e^{\frac{-2\pi k}{N}}$. The data sequence will process N+1 time to achieve the filter output as $y_k(n)$ for n=0, 1………N, where k is the frequency index (bin number) of interest. The DFT coefficient X(k) is the last datum from the Goertzel filter, that is:

$$X(k) = y_k(N) \quad (3)$$

Figure 2 shows the implementation of the Goertzel filter and presented by direct-form II realization. According to the direct-form II realization, we can write the Goertzel algorithm as follows [2]:

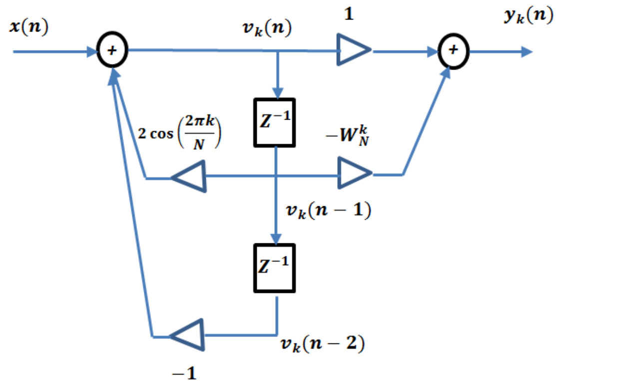

Fig 2. Second-order Goertzel IIR filter.

$$x(N) = 0 \qquad \text{For} \qquad n = 0,1 \ldots \ldots, N$$

$$v_k(n) = 2\cos\left(\frac{2\pi k}{N}\right) v_k(n-1) - v_k(n-2) + x(n) \quad (4)$$

$$y_k(n) = v_k(n) - W_N^k v_k(n-1) \quad (5)$$

The initial conditions $v_k(-2) = v_k(-2) = 0$

The squared magnitude $X(k)$ is computed as

$$|X(k)|^2 = v_k^2(N) + v_k^2(N-1) - 2\cos\left(\frac{2\pi k}{N}\right) v_k(N) v_k(N-1)$$
(6)

The Goertzel algorithm has the following advantages:

1. The Goertzel algorithm can be used to compute the DFT coefficient X(k) for a specified frequency bin k; unlike the fast Fourier transform (FFT) algorithm, all the DFT coefficients are computed once it is applied.

2. The operations avoid complex algebra. We need to process $v_k(k)$ N+1 times and then compute $|X(k)|^2$, if we want to compute the spectrum at frequency bin k.

Also, we can use the modified Goertzel filter showed in Figure 3, the corresponding transfer function is given by:

$$G_k(z) = \frac{V_k(z)}{X(z)} = \frac{1}{1 - 2\cos\left(\frac{2\pi k}{N}\right) z^{-1} + z^{-2}} \quad (7)$$

The squared magnitude of the DFT coefficient is given by

$$|X(k)|^2 = v_k^2(N) + v_k^2(N-1) - 2\cos\left(\frac{2\pi k}{N}\right) v_k(N) v_k(N-1)$$
(8)

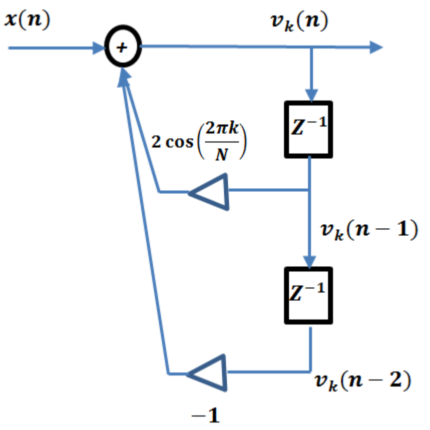

Fig 3. Modified second-order Goertzel IIR filter.

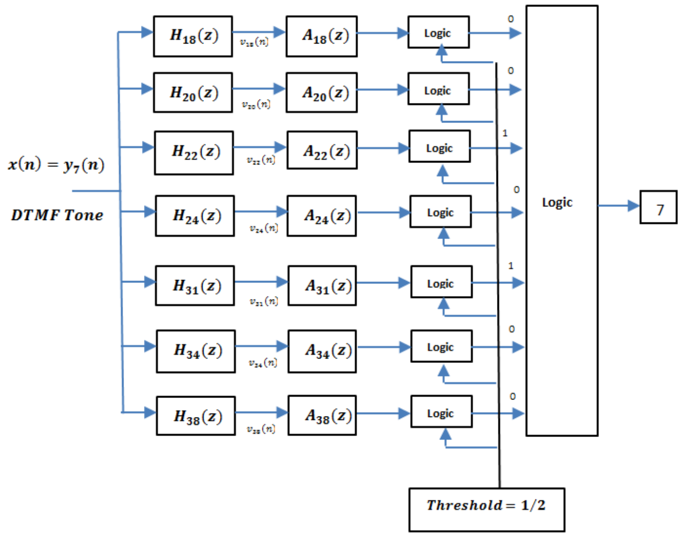

Fig 4. DTMF detectors using the Goertzel algorithm.

IV. DTMF Tone Detection Using the Modified Goertzel Algorithm

Based on the modified Goertzel algorithm and the specified frequencies of each DTMF tone shown in Figure 1, with the present of noise, we can develop the following design principles for DTMF tone detection [5]:

1. When the digitized DTMF tone x(n) is received, it has two nonzero frequency components from the following seven: 697, 770, 852, 941, 1,209, 1,336, and 1,477 Hz.

2. We can apply the modified Goertzel algorithm to calculate seven spectral values, which correspond to the seven frequencies in Figure 1. The single-sided amplitude spectrum is calculated as

$$A_k = \begin{cases} \frac{1}{N}\sqrt{|X(k)|^2}, & k = 0 \\ \frac{2}{N}\sqrt{|X(k)|^2}, & k = 1, \dots \dots N/2 \end{cases} \quad \textbf{(9)}$$

3. There is no complex algebra involved. Since the modified Goertzel algorithm is used, ideally, there are two nonzero spectral components. We will use these two nonzero spectral components to determine which key is pressed.

4. The frequency bin number (frequency index) can be determined based on the sampling rate fs, and the data size of N via the following relation:

$$k = \frac{f}{f_s} \times N \quad (10)$$

Given the key frequency specification in Table 1, we can determine the frequency bin k for each DTMF frequency with fs= 8000 Hz and N=205. The DTMF detector block diagram is shown in Figure 4.

5. The amplitude spectrum (A_k) has been normalized as following:

$$A_k = \frac{A_k}{max\,(A_k)} \qquad k = 1, \dots \dots N/2 \quad (11)$$

Such that the max value of amplitude spectrum equal to one. The threshold value can be considered as half. Note that there are only two nonzero spectral values; hence the threshold value should ideally be half of the individual nonzero spectral value. If the spectrum value is larger than the threshold value, then the logic operation outputs logic 1; otherwise, it outputs logic 0. Finally, the logic operation at the last stage is to decode the key information based on the 7-bit binary pattern.

Table 1 : DTMF Frequencies and Their Frequency Bins

DTMF Frequency (Hz)	Frequency Bin
697	18
770	20
852	22
941	24
1209	31
1336	34
1477	38

Detection of DTMF tones corrupted with various levels of Additive White Gaussian Noise (AWGN). For the experiment addition of AWGN to the DTMF tones was implemented using [6]:

$$n(t) = \sqrt{\alpha} \times r(t) \quad (12)$$

where α is the variance of the noise and *r(t)* is a vector of pseudo-random values, generated using Gaussian distributed random numbers with zero mean and one standard deviation. The noise term *n(t)* is linearly summed with *x(t)* for every DTMF input sample, to generate DTMF signals with AWGN.

V. The system model and simulation results.

V.I System model:

As we can see from Figure 5, the DTMF tone will be generated when one of keypad bottoms has been pressed, which generate two frequencies (High & Low). The signal will be send throw noisy channel, the receiver uses modified Goertzel algorithm to detect the tow frequencies with the present of noise

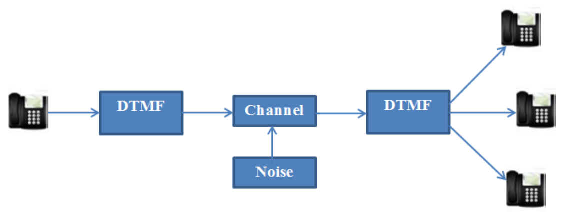

Fig 5. DTFM generation and detection model

V.II Simulation results:

We have performed these simulations using MATLAB. The Key Error Rate (KER) is the parameter which has been used to analyze the performance of the system. The KER is probability that any given key of the received data will be in error. The simulation results obtained by plotting the KER against the Signal to Noise Ratio (SNR). Different SNR for the DTMF input samples with different noise coefficient α was computed. Additionally, error rates were computed for all 205 sample validations, to determine the efficiency of the DTMF detection model. The overall system performance obtained for the noisy and filtered input samples is compared in Figure 6. Figure 6 shows the simulation results of the keys 1, and 9. The results show similar performance and the KER decreases as SNR increases.

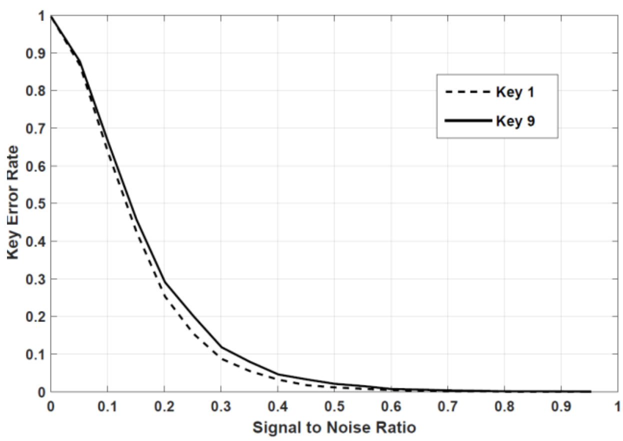

Fig. 6. Key Error Rate for Keys 1 and 9.

Figures 7-10 show the influence of the noise on the spectrum of the DTMF. The Key 3 is chosen as an example and consists of two frequencies (697 Hz, 1477 Hz). The spectrum of the Key 3 is shown for 4 cases. Figure 7, shows the spectrum when there is no influence of noise. Figure 8, 9, 10 shows the influence of the noise on the amplitude spectrum for different values of SNRs. SNR of 0.1 represents low value and SNR of 0.5 represents moderate value and SNR of 2 represents high value SNR. The Amplitude spectrum considered to be normalized value (maximum of 1), the threshold level (0.5) represents the occurrence of error. If any spectrum other than (697Hz and 1477Hz) exists above the threshold level means an occurrence of errors. From the figures, it is very clear that there are no errors when SNR=2.

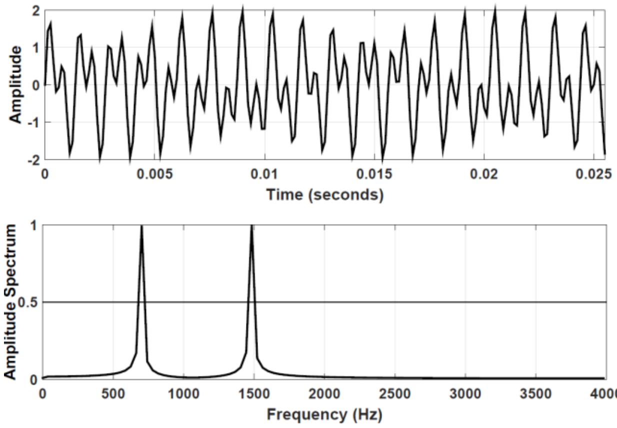

Fig. 7. Amplitude spectrum without noise.

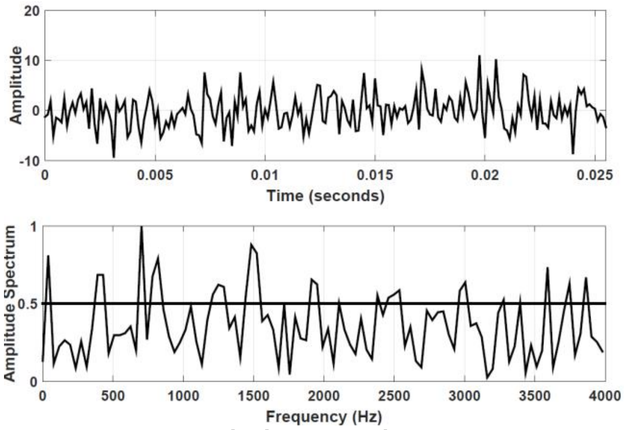

Fig 8. Amplitude spectrum when SNR=0.1

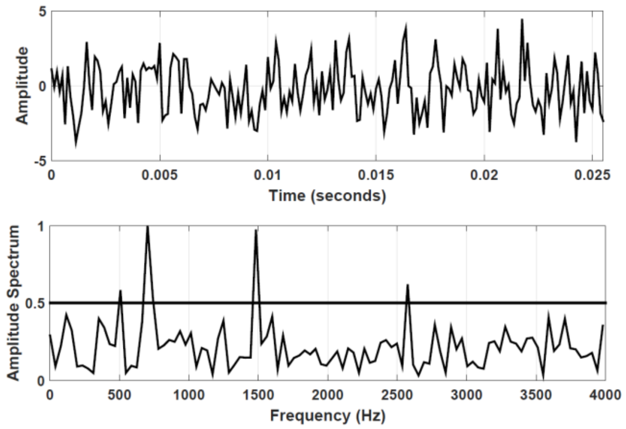

Fig 9. Amplitude spectrum when SNR=0.5

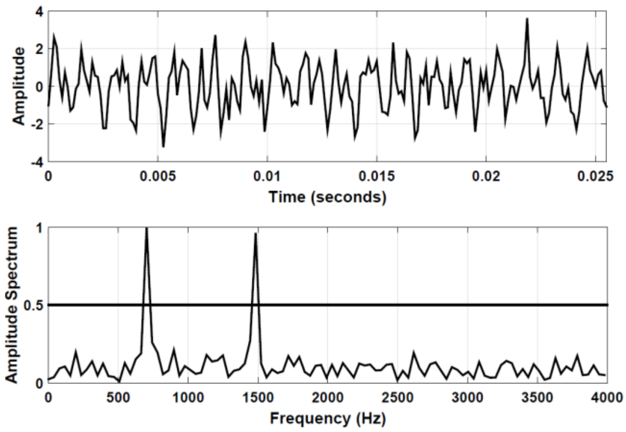

Fig 10. Amplitude spectrum when SNR=2.0

VI. Conclusion:

In this paper, an efficient detection of DTMF tones under the influence of AWGN and frequency variation is presented. The toolboxes in the MATLAB environment were used in the experiment for DTMF tone generation, signal analysis including noise reduction, and phone number detection. Experimental results obtained demonstrate the feasibility of applying Goertzel algorithm for DTMF tone detection.. In addition, different SNR has been applied on key 3 and show the frequency responses of the second-order Goertzel bandpass filters. It was observed that, as the SNR levels increase the KER begin to decrease gradually.

VII. References

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VIII. Biographies

Hamza Ali Eldenferia was born in Misurata /Libya, on July 21, 1985. He received Bsc degree in Electrical and Electronics from University of Misurata, in 2008. He got Msc degree in Network and Telecommunication Engineering from Curtin University /Australia in 2011. Moreover, he is currently lecturer in Electronics Engineering Department at College of Industrial Technology-Misurata-Libya.

Jamal Elbergali received his Bsc degree in 1990 from Tripoli University, Tripoli , Libya and Msc in 1993 from Aligarh Muslim University, India.. His Ph.D was received in 2003 from the University of Cape Town (UCT). Currently he is a staff member at the College of Industrial Technology. His research interests on the Evolved Packet System (EPS), and Long Term Evolution (LTE advanced).