

# LQG Control Design for Boiler-Turbine system

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*Abstract* – Designing controller for the Multi Input Multi Output (MIMO) process is difficult because of the changes in process dynamics and interactions between process variables. This paper presents the approach to design a Linear Quadratic Gaussian (LQG) Controller for a multivariable boiler-turbine process. In this approach a Linear Quadratic Regulator (LQR) with integral action and Kalman’s state estimation techniques is used to control the plant. The performance of the proposed system is tested and compared with PID results for reference tracking and disturbance rejection behavior using MATLAB/Simulink simulations. These results show that the suggested multivariable controlled technique outperforms the existing PID-controller in many aspects improving the control performance significantly and yielding much tighter reference value tracking during load change.

*Index Terms*—**Multivariable Control, Linear Quadratic Regulator, Kalman Filter, PID controller.**

## I. INTRODUCTION

The power plant is a highly complex, nonlinear, and time varying system, where its control is of a multi-loop nature with interactions between different loops. Because of these interactions between input and output variables, it is very complex to design suitable controller for MIMO systems. Several control techniques are available to handle multivariable systems. Multivariable control problems are traditionally solved by centralized Proportional Integral Derivative (PID) controllers to obtain the desired overall control function. However, Decentralized PID control has been widely used in MIMO industrial processes due to the simplicity in implementation and loop failure tolerance of the resulting control system. In this configuration, the MIMO system is divided into individual SISO PID loops and tuned mainly on a single loop basis. To compensate the interactions between variables, decentralized controllers can be employed by designing suitable decoupler so that the MIMO system is decoupled into several SISO systems and can be controlled using simple feedback controllers. However, additional restrictions will be introduced in the feedback properties of system with decoupler [1]. Non-minimum phase behaviour is one of the challenges in MIMO systems because it will lead to inverse response. It requires proper input output pairing

of variables using Relative Gain Array (RGA) to handle the process with non-minimum phase response [2].

Different methods are used to tune the decentralized PID controllers such as the Biggest Log Modulus (BLT) [3], a relay feedback auto-tuning technique based in the Ziegler-Nichols method for closed loop systems [4] and IMC method [5].

Various advanced methods have also been used to obtain optimum PID parameters, such as genetic algorithms [6], particle swarm optimization [7], and multi-objective robust techniques based on linear matrix inequalities (LMIs) [8]. However, these methods have problems of either enormous computation efforts or difficulty in tackling non-convex optimization problems.

In the case of large changes in operating conditions, effective control systems must be developed to have an appropriate performance of the boiler-turbine units. Various control methods have been used for boiler or boiler-turbine units. In the early works, decoupling controller [9] for performance control of boiler-turbine units have been designed. Also, multivariable predictive control based on local model networks [10], fuzzy based control systems for thermal power plants [11, 12], neuro-fuzzy network modelling and PI control of a steam-boiler system [13] have been presented. We can also mention robust control using  $H^\infty$  mixed-sensitivity approach [14].

Even though the above mentioned approaches can manage to improve the control performance, as observed within different simulation environments, it can lack many practical aspects such as online tuning and ease of implement within a real plant distributed control system (DCS).

The control methodologies discussed in this paper are based on state space approach and the decentralized PID controller is taken for comparison purpose. The controller used is Linear Quadratic Gaussian Compensator LQG with integral action. The achievement in this study is to add an integral action to enable the controller to track a desired setpoint. In LQG controller design, plant is considered as a stochastic process or nondeterministic process, where the process is affected by the process noise and the measurement noise.

The steps for designing the LQG controller is given as follows: First the linearized mathematical model of the boiler-turbine system around an operating points is derived; then the controllability and observability of the system is checked and then state feedback vector is obtained using LQR with integral action to handle the tracking problem. Next a full state observer is developed with Kalman filter and finally by combining these two to form the LQG controller.

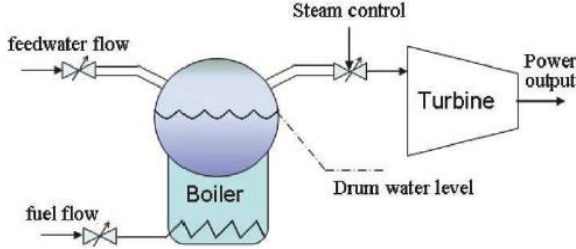


Figure 1. A schematic of a boiler-turbine unit

## II. PROCESS MODELLING

The boiler and steam turbine are the main elements in a power generation plant. The purpose of the boiler is to produce steam at high pressure and temperature which is driven to the turbine to convert the heat to mechanical energy. The turbine is connected to the generator to produce electricity. In the turbine the steam passes through a nozzle which expands the gas increasing the velocity and producing the rotational movement.

One of the first steps for control engineers is to design a model of the process to control. The model should accurately describe the dynamics of the system to understand its behavior and allow the operators to design the most appropriate control. Boiler systems for power plant applications consist of hundreds of local control loops. When designing the model only some of them are used, considering the rest fast and accurate to be avoided. The boiler-turbine model used in this paper was first developed by Bell and Astrom [15] and has been popularly adopted in validating various controllers for the boiler-turbine system in simulation. The parameters were estimated from the data collected from the Synvendska Kraft AB Plant in Malmo, Sweden. The rate power of the plant is 160 MW. A schematic diagram of the boiler-turbine unit is shown in Fig. 1. A simplified mathematic model of the system can be described by nonlinear differential equations of the form:

$$\dot{x}_1 = -0.0018u_2x_1^{\frac{9}{8}} + 0.9u_1 - 0.15u_3 \quad (1)$$

$$\dot{x}_2 = (0.073u_2 - 0.016)x_1^{\frac{9}{8}} - 0.1x_2 \quad (2)$$

$$\dot{x}_3 = \frac{(141u_3 - (1.1u_2 - 0.19)x_1)}{85} \quad (3)$$

where state variables  $x_1$ ,  $x_2$  and  $x_3$  represent drum pressure ( $\text{kg/cm}^2$ ), electric output in megawatts (MW), and fluid density ( $\text{kg/m}^3$ ), respectively. As shown on Fig.1, the control inputs  $u_1$ ,  $u_2$  and  $u_3$  are the valve positions of fuel flow, normalized steam flow control, and normalized feed water flow, respectively. Denote  $x=[x_1 \ x_2 \ x_3]^T$  and  $u=[u_1 \ u_2 \ u_3]^T$ . The outputs of the plant are given by

$$y_1 = x_1 \quad (4)$$

$$y_2 = x_2 \quad (5)$$

$$y_3 = 0.05(0.13073x_3 + 100a_{cs} + \frac{q_e}{9} - 67.975) \quad (6)$$

Where  $y_3$  is the drum water level (m),  $a_{cs}$  and  $q_e$  are steam quality and evaporation rate (kg/s), respectively. They are given by

$$a_{cs} = \frac{(1 - 0.001538x_3)(0.8x_1 - 25.6)}{x_3(1.0394 - 0.0012304x_1)}$$

$$q_e = (0.854u_2 - 0.147)x_1 + 45.59u_1 - 2.514u_3 - 2.096$$

Due to actuator limitations, all the control inputs are subject to the following constraints:

$$\begin{aligned} 0 &\leq u_i \leq 1 & i &= 1, 2, 3 \\ -0.007 &\leq \dot{u}_1 \leq 0.007 \\ -2 &\leq \dot{u}_2 \leq 0.02 \\ -0.05 &\leq \dot{u}_3 \leq 0.05 \end{aligned} \quad (7)$$

There are several typical operating points of Bell and Astrom model [15], but the linear control design for the unit found in literature usually takes the linearized model at the operating point  $x_0=[108 \ 66.65 \ 428]^T$ ,  $u_0=[0.34 \ 0.69 \ 0.433]^T$  and  $y_0=[108 \ 66.65 \ 0]^T$ . the nonlinear model is simulated using S-function/Simulink as listed in the Appendix, then linearized around the operating point using Linmod /Matlab function. The Linearized model is given by the following state-space model.

$$\begin{aligned} A &= \begin{bmatrix} -0.0025 & 0 & 0 \\ 0.0694 & -0.1 & 0 \\ -0.0067 & 0 & 0 \end{bmatrix} & B &= \begin{bmatrix} 0.9 & -0.349 & -0.15 \\ 0 & 14.16 & 0 \\ 0 & -1.398 & 1.659 \end{bmatrix} \\ C &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.0063 & 0 & 0.0047 \end{bmatrix} & D &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.253 & 0.512 & -0.014 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \quad (8)$$

### III. DECENTRALIZED PID TUNING

In this study, a relay feedback auto-tuning method [4] for closed loop systems was used to tune PID parameters. In this method, the controller is replaced by a relay which induces a sustained oscillation. The frequency of this oscillation ( $P_u$ ) and its amplitude ( $A$ ) as shown in Fig.2 can be used to determine the parameters of the PID controller based on Ziegler-Nichols tuning rules. The advantages of relay feedback technique are the tuning can be done online and has a less chance of getting the system unstable during the tuning of PID controller, since it is a closed-loop method. The ultimate gain can be computed as:

$$K_{cu} = \frac{4h}{\pi A} \quad (9)$$

where  $h$  is the height of the relay and  $A$  is the amplitude of oscillation as shown in Fig. 2.

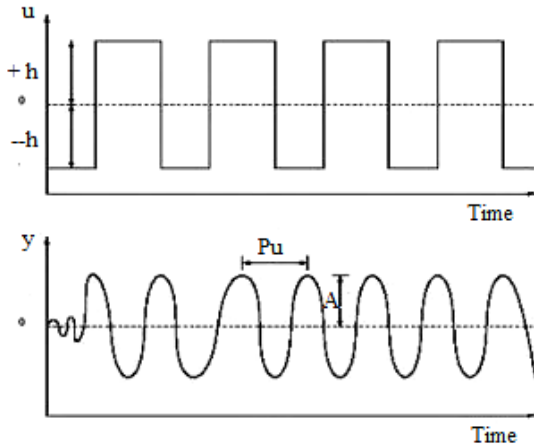


Figure 2: Closed-loop response of Relay feedback

Having determined the ultimate gain  $K_{cu}$  and the oscillation period  $P_u$ , the PI controller tuning parameters can be obtained using Ziegler-Nichols or Tyreus-Luyben rules as listed in Table 1 [16]. These parameters are used to establish the correction  $u(t)$  from error  $e(t)$  via the equation:

$$u(t) = K_p \left( e(t) + \frac{1}{T_i} \int_0^t e(t) dt \right) \quad (10)$$

TABLE 1: ZIEGLER-NICHOLS AND TYREUS-LUYBEN TUNING RULES

Controller method	$K_p$	$T_i$
Ziegler-Nichols	$K_{cu} / 2.2$	$P_u / 1.2$
Tyreus-Luyben	$0.31 K_{cu}$	$2.2 P_u$

In relay auto-tuning for MIMO control system, the relay feedback method is usually applied using a

sequential design strategy. This strategy involves closing each loop once it is tuned, until all the loops are covered. The tuning sequence should be repeated in an iterative manner to account for the effect of loop interactions. Faster convergence can be achieved when the fast loop is tuned first [16]. However, for unstable control systems it is better to close the unstable loop first in order to cope with instabilities.

### IV. LINEAR QUADRATIC REGULATOR (LQR) DESIGN

Linear Quadratic Regulator (LQR) is an optimal multivariable state feedback control approach that minimizes the excursion in state trajectories of a system while requiring minimum controller effort. The principle of a LQR controller is given in Fig.3.

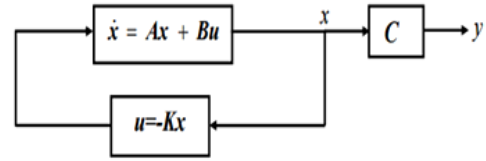


Figure 3: Linear Quadratic Regulator Structure.

This regulator provides an optimal control law for a linear system with quadratic performance index yielding a cost function on the form

$$J = \int_0^{\infty} [x^T(t)Qx(t) + u(t)^TRu(t)] dt \quad (11)$$

where  $Q$  and  $R$  are weighting parameters matrices that penalize the states and the control effort, respectively. These matrices are therefore controller tuning parameters. A first choice for the matrices  $Q$  and  $R$  in (10) is given by the Bryson's rule [17]. Although Bryson's rule sometimes gives good results, often it is just the starting point to a trial-and-error iterative design procedure aimed at obtaining desirable properties for the closed-loop system.

In a LQR design, because of the quadratic performance index of the cost function, the system has a mathematical solution that yields an optimal control law

$$u(t) = -Kx(t) \quad (12)$$

where  $u$  is the control input and  $K$  is the gain given as

$$K = R^{-1}B^T S \quad (13)$$

where  $S$  is the solution of the algebraic Riccati Equation

$$SA + A^T S + Q - PBR^{-1}B^T S = 0 \quad (14)$$

### V. KALMAN FILTER STATE ESTIMATOR

As mentioned for the case of the LQR controller, all sensors for measuring the different states are assumed to be available. This is not a valid assumption in practice. Therefore, it is important to incorporate a state observer to estimate the non-measurable states. Kalman filter can be applied to estimate the system state in the presence of noise. Block schematics of a Kalman filter connected to the plant is shown in Fig. 4. The Kalman filter is essentially a set of mathematical equations that

implement a predictor-corrector type estimator that is optimal in the sense that it minimizes the estimated error covariance when some presumed conditions are met.

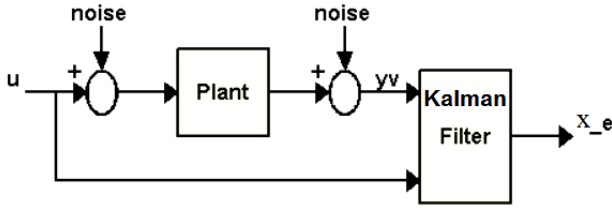


Figure 4: Kalman filter used as an optimal observer

The mean square estimation error is given by

$$J = E[(x(t) - \hat{x}(t))^T (x(t) - \hat{x}(t))] \quad (15)$$

The optimal gain Kalman gain is given by

$$L = S_e C^T R^{-1} \quad (16)$$

Where  $S_e$  is the steady-state solution of the following algebraic Riccati equation

$$0 = S_e A^T + A S_e + Q_n - S_e C^T R_n^{-1} C S_e \quad (17)$$

where  $Q_n$  and  $R_n$  are the process and the process and measurement noise covariance, respectively. Tuning of the Kalman filter are required if these are not known. Then, the state observer is calculated as

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - C\hat{x}(t)) \quad (18)$$

## VI. LQG CONTROLLER WITH INTEGRAL ACTION

Combining a Kalman filter with LQR feedback results in a very robust linear quadratic Gaussian regulator (LQG) controller. In LQG, we replace the state vector  $x$  in LQR by its estimation  $\hat{x}$  obtained from Kalman filter as shown in Fig. 5.

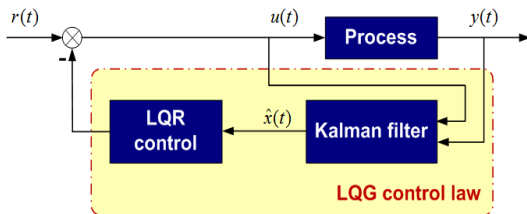


Figure 5: block diagram of LQG

Normal LQR cannot track any given set points. There are several ways to solve the problem such as to add an integrator [18]. In tracking (trajectory following) systems, we require that the output of a system track or follow a desired trajectory in some optimal sense. However, in the normal LQR regulator the desired trajectory is simply the zero state or the equilibrium operating points of linearization. In this study, LQG controller with integral action is used for the considered boiler-turbine as shown in Fig. 6. The role of the integral action to cancel the tracking error defined as  $r-y$ .

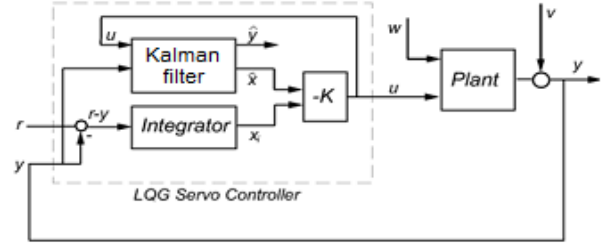


Figure 6: LQG controller with integral action.

The basic approach in the integral feedback is to create a state within the controller that computes the integral of the error signal. This is done by augmenting the system model with a new state  $x_i$  as follows

$$\dot{x}_i = r - y \quad (19)$$

Combining the state space model with the new state to form the following augmented state equation

$$\begin{bmatrix} \dot{x} \\ \dot{x}_i \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ x_i \end{bmatrix} + \begin{bmatrix} B \\ -D \end{bmatrix} u$$

$$y = [C \ 0] \begin{bmatrix} x \\ x_i \end{bmatrix} + Du \quad (20)$$

$$u = -K[x; x_i]$$

## VII. SIMULATIONS AND RESULTS

In this study, the decentralized PI controller is used as a comparison with the LQG controller. As shown in Fig. 7, the relay feedback method described in section 3 is applied to the MIMO decentralized PI controller for the boiler-turbine system using Ziegler-Nichols and Tyreus-Luyben tuning rules. Starting with the first loop, the controller parameters converge in 3 relay-feedback tests. Table 2 shows the results of the ultimate gain ( $K_{cu}$ ) and the ultimate frequency ( $P_u$ ) for the three loops.

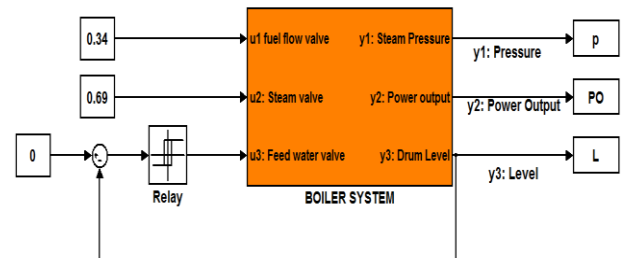


Figure 7: Relay tuning configuration for level loop.

TABLE 2: ULTIMATE GAIN AND ULTIMATE FREQUENCY

	Level loop	Power loop	Pressure Loop
$K_{cu}$	15.91	1.02	1.30
$P_u$	40.82	11.38	3.32

These parameters and the rules listed in Table 1 are used to calculate the PI tuning parameters.

The comparison between PI tuning using Tyreus-Luyben and Ziegler-Nichols methods to set-points changes are sit it is clearly shown from Fig.8 and Fig.9 that the Ziegler-

Nichols method gives an aggressive response compared to Tyreuse-Luyben method.

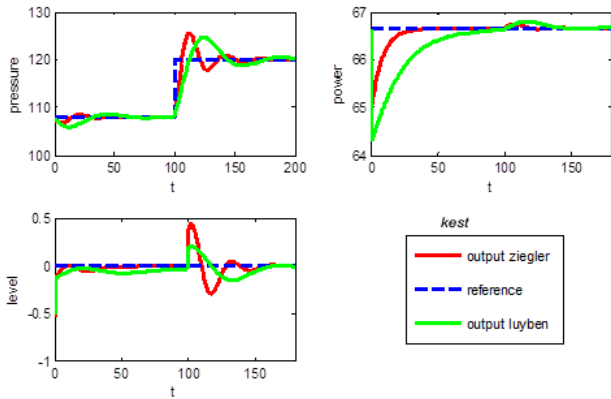


Figure 8: Ziegler and Luyben output responses comparison.

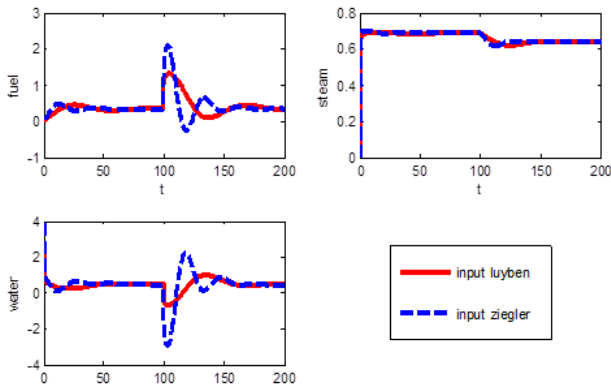


Figure 9: Ziegler and Luyben input responses comparison.

The simulation diagram for the LQG with integral action using Simulink is shown in Fig. 10. The design parameters of LQG Q and R are selected using try and error tuning method and given as follows:  
 $R = \text{diag}([2.4e5, 1.8e5, 9e4])$  and  $Q = \text{diag}([2, 2, 2, 20, 24, 6e3])$ .

After the measurement and disturbance noise covariance matrices are determined, the MATLAB function kalman was used to find the optimal Kalman gain L and the covariance matrix P. Fig. 11 shows the comparison of the system states with noises and the estimated states. It is clearly shown that Kaman filter is able to estimate the system states when there are noises in the system.

Fig. 12 and Fig. 13 show the responses of the controlled outputs and the manipulated inputs, respectively for both LQG and PID controllers. In this simulation the system is tested for the system responses to step changes in boiler pressure set-point changes at  $t = 100(\text{sec})$ . From these results it is clear that the LQG controller has good tracking performance. In addition, this test proves that the method of LQG is the best compared to the PID method in terms of less overshoot, less aggressive and best disturbance.

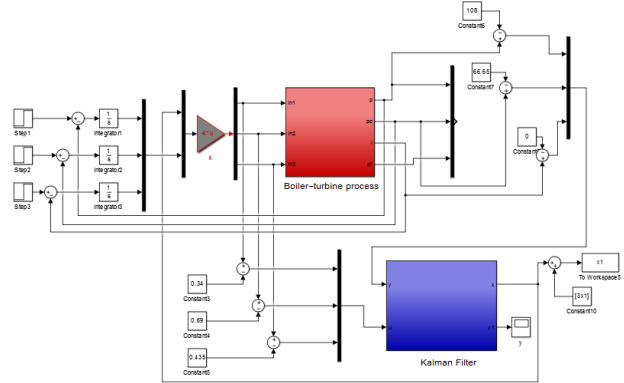


Figure 10: Simulink simulation of LQG

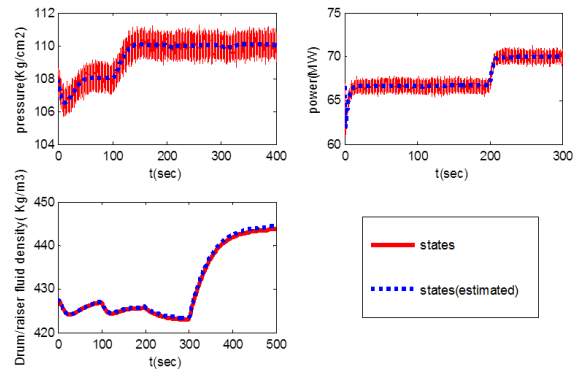


Figure 11: comparison between system states and the estimated states

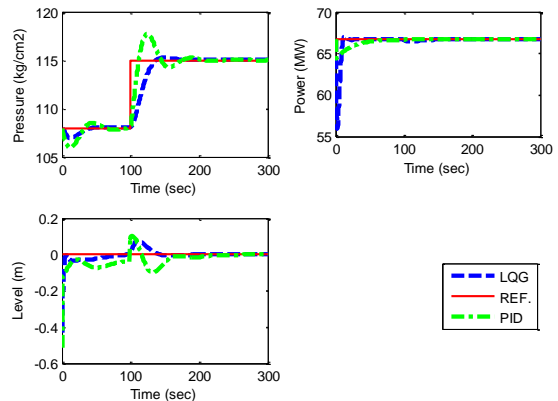


Figure 12: Output responses to set point changes of LQG and PID

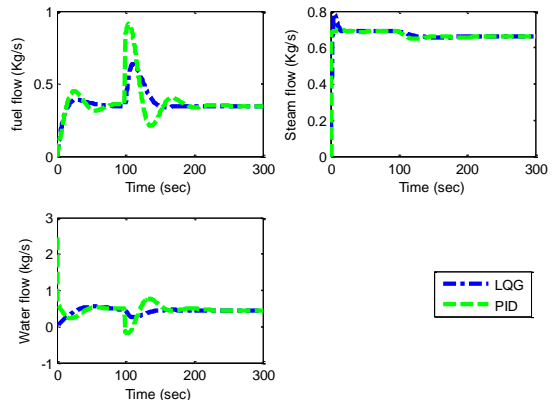


Figure 13: Inputs responses to set point changes of LQG and PID

## CONCLUSION

In this study an optimal LQG controller to control the boiler-turbine system was discussed. KALMAN filter was used as linear observer to estimate the system states. In the normal LQR regulator the desired trajectory is simply the zero state or the equilibrium operating points of linearization. In this paper LQG with integral action was used for set-point tracking to maintain the output as close as possible to the desired trajectory. Results of LQG with set-point tracking method are better compared to the PID methods in terms of overshoot and disturbance rejections.

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## APPENDIX

The s-function Simulink program used to simulate the boiler-turbine system is listed below:

```
function [sys,x0]=turbine2(t,x,u,flag)
if flag == 0
sys=[3,0,3,3,1,1];
x0 = [108 66.65 428];      % Initial states
elseif (flag) == 1
sys(1)= (-0.0018*u(2)*x(1)^(1.125))+0.9*u(1)-0.15*u(3);
sys(2)=(0.073*u(2)-0.016)*x(1)^(1.125)-(0.1*x(2));
sys(3)= (141*u(3)-(1.1*u(2)-0.19)*x(1))/85;
elseif flag == 3
sys(1)=x(1);
sys(2)=x(2);
asc=(1-0.001538*x(3))*(0.8*x(1)-25.6)/(x(3)*(1.0394-0.0012304*x(1)))
qe=(0.854*u(2)-0.147)*x(1)+45.59*u(1)-2.514*u(3)-2.096;
l=0.05*(0.13073*x(3)+100*asc+0.111*qe-67.975);
sys(3)=l;
else
sys=[];
end
```