Adaptive (LQG) Controller of Geostationary Satellite Model using Discrete State-Space Technique

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***Abstract* - The main objective of this paper is to study and analyse an adaptive linear quadratic Gaussian (LQG) controller applied to single-axis geostationary satellite attitude control system. The controller was examined under different sudden load changes. Moreover, a new algorithm for adaptive LQG controller based on discrete state-space technique has been developed. The algorithm was enhanced by an on-line Kalman filter to overcome certain limitations existing in previous designs of linear quadratic controllers (LQC). The on-line Kalman filter is a software sensor used to ensure a robust performance when the states are difficult to sense by using conventional hardware sensors. In addition, the integral action of PID controller was incorporated into LQG controller to ensure zero steady-state error. This methodology provides the user with the advantages of both PID and LQG controllers. Simulation experiments (using matlab software) were carried out to demonstrate the effectiveness of the modified adaptive linear quadratic Gaussian (ALQG) controller on the closed loop system performance. The controller was applied to geostationary satellite simulated model. Encouraging results were achieved and adaptive (LQG) controller could be examined other applications.**

***Index Terms* - adaptive Control, Discrete State-space LQG Design, On-Line Kalman filter, LQG Control, geostationary satellite System.**

I Introduction

In spite of developed modern control techniques like fuzzy logic controllers or neural networks controllers, PID controllers constitute an important part at industrial control systems. Thus, improvement of PID design and implementation methodology has a series potential to be used in industrial engineering applications. The reason for the popularity of PID controllers lies in their remarkable effectiveness in regulating a wide range of processes, simplicity of their structures and their ability to regulate steady-state error to zero [1]. However, PID controllers need to be retuned when the systems to be controlled is subjected to significant changes in order to achieve satisfactory performance. These changes could be caused by large time-delays, time-varying dynamics, large non-linearity, or non-negligible disturbances). For this reason, during the last two decades much work in linear control theory devoted to design adaptive controllers with a structure similar to that of PID controllers [2]. Adaptive controllers could be designed via transfer function approach or state-space technique. The main drawback of most adaptive linear quadratic Gaussian controllers that are based on transfer function approach is the use of polynomial algorithms which are slow to emerge. In contract, the linear algebraic tools that are utilised in state-space technique are more advanced and more suitable for optimal control designs. Therefore, the state-space technique is preferred over the transfer function approach, especially for multivariable and non-linear systems [3,4]. The main contribution of this paper is to develop a new state-space adaptive linear quadratic Gaussian framework. In order to assess the performance of the closed-loop system, the controller is applied to satellite model.

II Satellite Model

Geostationary satellite usually requires attitude control so that antennas, sensors, and solar panels are properly oriented. For example, antennas are usually pointed towards a perpendicular location on the earth, while solar panels need to be oriented towards the sun for maximum power generation. To gain insight into the full three-axis attitude-control system, we often consider one axis at a time. Figure (1) depicts this case where motion is allowed only about an axis perpendicular to the page. When a geostationary satellite body is accelerated, internal elastic deflections are always present. If these internal deflections are negligibly small relative to the gross motion of entire satellite body, the geostationary satellite body is called rigid satellite body. Physically, the moment of inertia $J\ (Kg.m^2)$ of a rigid geostationary satellite body is a measure of its resistance to angular acceleration. The geostationary satellite is assumed to be frictionless environment [5,6].

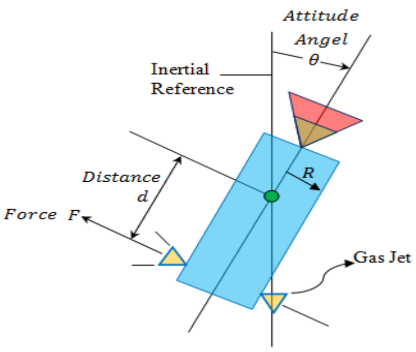

Figure (1): Geostationary satellite control schematic

The equations of motion of the satellite system are given by:

$$J\ddot{\theta} = M_C + M_D \quad (1a)$$

$$M_C = F \times d \quad (1b)$$

$$J = \frac{1}{2} \times MR^2 \quad (1c)$$

where F is the force coming from the reaction jet, d is the distance of the body from its mass center, J is the moment of inertia of the satellite about its mass centre, M_C is defined as the control torque applied by the thrusters which comes from the reaction jet, M_D is the disturbance torque, M is the mass of satellite, R satellite radius and $\theta\ (rad)$ is the angle of the satellite axis with respect to an "inertial" reference. Normalizing, we define:

$$u(t) = {}^{M_C}/_{J} \quad (2)$$

$$\xi'(t) = {}^{M_D}/_{J} \quad (3)$$

The dynamic equation (1) becomes:

$$\ddot{\theta}(t) = \left(u(t) + \xi(t)\right) \quad (4)$$

where $u(t)$ is the control input signal and $\xi(t)$ is the process noise.

Taking Laplace Transform of equation (4):

$$\theta(s) = \frac{1}{s^2}\left[U(s) + \xi(s)\right] \quad (5)$$

The discrete model of the satellite system can be written as :

$$\theta(z) = \frac{T_s^2}{2}\frac{(z+1)}{(z-1)^2}\left[U(z) + \xi(z)\right] \quad (6)$$

A discrete state-space of Single-Axis Geostationary Satellite attitude control model at $T_s = 1sec$ is:

$$\begin{bmatrix} X_1(t+1) \\ X_2(t+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} + \begin{bmatrix} 0.5 \\ 1.5 \end{bmatrix} u(t) + \begin{bmatrix} 0.5 \\ 1.5 \end{bmatrix} \xi(t) \quad (7a)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} + v'(t) \quad (7b)$$

In this model, the state $X_1(t)$ is the position state (rad) of satellite, and the state $X_2(t)$ is the velocity state of satellite (rad/sec). However, in this work the proposed controller is used to control the position of one axis of the satellite model.

III The New Adaptive LQG Controller Algorithm

This control design is an extension to the previous works of [3,4].Figure (2) shows the adaptive linear quadratic Gaussian with reference signal $r(t)$ and integral control block.

In Figure (2), the estimated dc gain $N(\hat{\theta})$ is introduced on-line into the design in the presence of reference signal $r(t)$ to ensure that the system output signal $y(t)$ is equal to the reference signal $r(t)$ in the steady-state. The vector $\hat{\theta}$ represents estimated satellite plant parameters.

$$\hat{\theta} = \left[-\hat{a}_1 - \hat{a}_2 .. -\hat{a}_{n_a} \quad \hat{b}_0\ \hat{b}_1\ \hat{b}_2 .. \hat{b}_{n_b} \quad \hat{c}_0\ \hat{c}_1\ \hat{c}_2 .. \hat{c}_{n_c}\right]^T$$

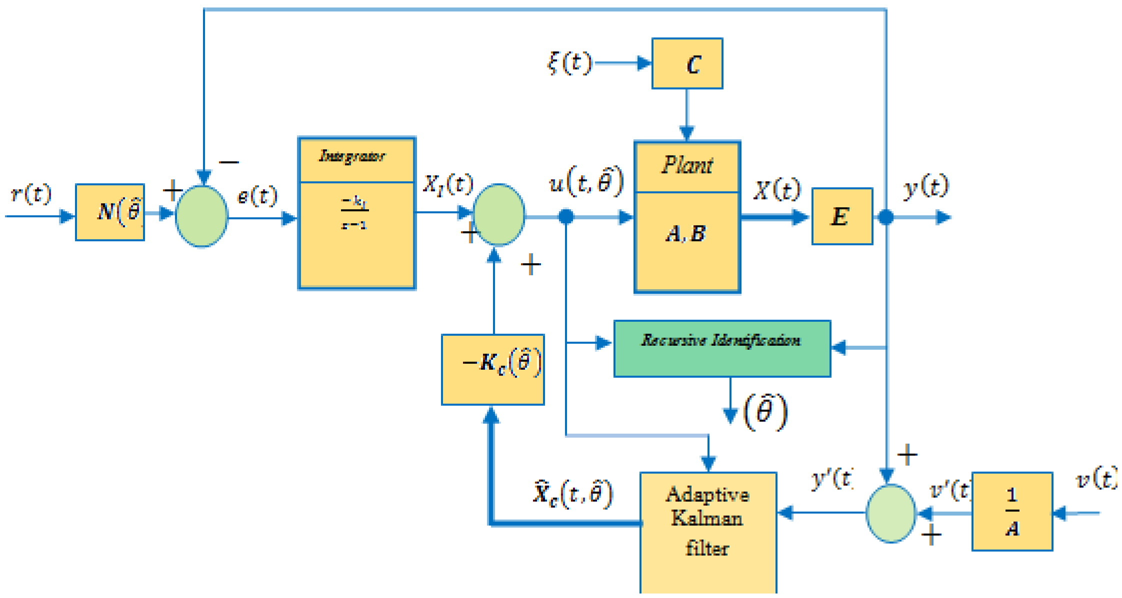

Figure (2) Adaptive Linear Quadratic Gaussian controller with integral action

A discrete state-space model of any system can be presented in a discrete matrix-vector equation as follows:

$$X(t+1) = \boldsymbol{A}X(t) + \boldsymbol{B}u(t) + \boldsymbol{C}\xi^{'}(t)$$
$$y(t) = \boldsymbol{E}X(t) + b_0u(t) + \xi^{'}(t) + v'(t)$$

where $X(t)$ and $X(t+1)$ is the $n \times 1$ state vectors, $u(t)$ is the control input signal, $y(t)$ is the system output signal, $\xi^{'}(t)$ defined as the process noise and $v'(t)$ is Measurement Noise. $\boldsymbol{A}$ is the $n \times n$ system dynamic matrix, $\boldsymbol{B}$ is the $n \times 1$ input matrix, $\boldsymbol{E}$ is the $1 \times n$ output matrix, $\boldsymbol{C}$ is the $n \times 1$ noise matrix, n is the order of the system and (t) equals to kT_s, k denotes the sampling instants or $k = 1,2,3,\cdots, T_s$ means the sampling time.

The values of both control input signal $u(t)$ and system output signal $y(t)$ are read every sampling instant in order to estimate satellite plant parameters $\hat{\theta}$.

In Figure (2), the adaptive gain N $(\hat{\theta})$,which can be obtained from the Recursive Least Squares estimator, is introduced in order to ensure that the output tracks the reference signal r (t) (i.e. y (t) = r(t) at steady state region).

In order to design ALQG controller, the estimated plant parameters $\hat{\theta}$, which are calculated by RLS, are used to formulate the new identified state-space model as:

$$\boldsymbol{x}_c(t+1,\hat{\theta}) = \boldsymbol{A}_c(\hat{\theta})\boldsymbol{x}_c(t,\hat{\theta}) + \boldsymbol{B}_c(\hat{\theta})u(t,\hat{\theta}) + \boldsymbol{C}_c(\hat{\theta})\xi($$
$$\boldsymbol{y}_c(t,\hat{\theta}) = \boldsymbol{E}_c(\hat{\theta})\boldsymbol{x}_c(t,\hat{\theta}) + \boldsymbol{B}_c(\hat{\theta})u(t,\hat{\theta}) + \xi(t) + v'(t)$$

(8b)

where

$$\boldsymbol{A}_c(\hat{\theta}) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\hat{a}_{na} & -\hat{a}_{na-1} & -\hat{a}_{na-2} & \cdots & -\hat{a}_1 \end{bmatrix},$$

$$\boldsymbol{B}_c(\hat{\theta}) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

$$\boldsymbol{C}_c(\hat{\theta}) = [\hat{b}_{nb} \quad \hat{b}_{nb-1} \cdots\cdots \hat{b}_1],$$
$$\boldsymbol{E}_c(\hat{\theta}) = [\hat{C}_{nc} \quad \hat{C}_{nc-1} \cdots\cdots \hat{C}_1]$$

and $v'(t)$ is the Measurement Noise.

The next step is to choose the user-defined gain R and the user pre-specified matrix Q to be positive definite to insure that the controller is converges to stability conditions.

At this stage, the adaptive Riccati equation expressed by the following equation:

$$P(\hat{\theta}) = Q + \boldsymbol{A}_c^T(\hat{\theta})\Big(P(\hat{\theta}) - P(\hat{\theta}) \times \boldsymbol{B}_c(\hat{\theta}) \times \big(R + \boldsymbol{B}_c^T(\hat{\theta}) \times P(\hat{\theta}) \times \boldsymbol{B}_c^T(\hat{\theta})\big) \times \boldsymbol{B}_c^T(\hat{\theta}) \times P(\hat{\theta})\Big)$$

(9)

The adaptive p matrix can be used to evaluate the needed adaptive gain matrix K$(\hat{\theta})$ as follows:

$$K_c(\hat{\theta}) = (R + B_c^T(\hat{\theta})P(\hat{\theta})B_c(\hat{\theta}))^{-1}.B_c^T(\hat{\theta}).P(\hat{\theta}).A_c(\hat{\theta}) \quad (10)$$

The gain N$(\hat{\theta})$ can be calculated as follows:

$$(N(\hat{\theta}))^{-1} = -\boldsymbol{E}_c(\hat{\theta})\big(\boldsymbol{A}_c(\hat{\theta}) - \boldsymbol{B}_c(\hat{\theta})K_c(\hat{\theta}) - I\big)^{-1} * \boldsymbol{B}_c(\hat{\theta})$$

As seen from above Figure, the integral block is added to the adaptive linear quadratic Gaussian controller in order to enable the controller to eliminate both external disturbance and steady-state error.

The integral state $X_I(t)$ represents the integral of the error $e(t) = y(t) - r(t)$.

The discrete integral is simply a summation of all past values of $e(t)$, which results in the difference equation:

$$X_I(t+1) = X_I(t) + e(t) = X_I(t) + \boldsymbol{E}X(t) - r(t) \quad (12)$$

Figure (2) shows that the control-law can be written as:

$$u(t,\hat{\theta}) = \boldsymbol{N}(\hat{\theta})r(t) - [K_I \quad \boldsymbol{K_c}(\hat{\theta})]\begin{bmatrix} X_I(t) \\ \hat{X}_c(t,\hat{\theta}) \end{bmatrix} \quad (13)$$

Where:

$u(t,\hat{\theta})$: optimal adaptive control signal.

$\boldsymbol{N}(\hat{\theta})$: adaptive gain.

$r(t)$: reference signal.

$\boldsymbol{K_c}(\hat{\theta})$: adaptive gains matrix.

$\hat{X}_c(t,\hat{\theta})$: adaptive estimated system states.

$X_I(t)$: the integral state.

K_I : integral gain.

$\mathrm{P}(\hat{\theta})$: adaptive positive definite lapanove matrix.

The adaptive time-varying Kalman filter is a generalization of the steady-state filter for time-varying systems or LTI systems with non stationary noise covariance. Given the plant state and measurement equations as:

$$\boldsymbol{X}(t+1) = \boldsymbol{A}\boldsymbol{X}(t) + \boldsymbol{B}u(t) + C\xi(t)$$
$$y(t) = \boldsymbol{E}X(t) + \beta_0 u(t) + \xi(t) + v'(t)$$

where, $X(t)$ and $X(t+1)$ is the $n \times 1$ state vector, $u(t)$ is the control input signal, $y(t)$ is the system output signal, $\xi(t)$ defined as process noise, $v(t)$ is the measurement noise; A is the $n \times n$ system matrix, B is the $n \times 1$ input matrix, E is the $1 \times n$ output matrix; C is the $n \times 1$ disturbing matrix, n is the order of the system and (t) equals to $k' T_s$, k' denotes the sampling instants or $k' = 1,2,3,\cdots$, T_s means the sampling time.

The adaptive time-varying Kalman filter is given by the following recursions:

Firstly, the measurement update period in which the estimation of the states, evaluating the covariance matrixes (P $(\hat{\theta})$ and $\mathrm{M}(\hat{\theta})$) are performed at current time depending on the values of (estimated states, $\mathrm{P}(\hat{\theta})$ and $\mathrm{M}(\hat{\theta})$) in previous time. The measurement update equations can be arranged as follows:

$$\mathrm{M}(\hat{\theta}) = \mathrm{P}(\hat{\theta}) * \boldsymbol{E_C}'(\hat{\theta}) / (\boldsymbol{E_c}(\hat{\theta}) * \mathrm{P}(\hat{\theta}) * \boldsymbol{E_C}'(\hat{\theta}) + \mathrm{R}) \quad (14a)$$

$$\hat{\boldsymbol{X}}_c(t,\theta) = \hat{\boldsymbol{X}}_c(t,\hat{\theta}) + \mathrm{M}(\hat{\theta}) * \left(y(t) - \boldsymbol{E_c}(\hat{\theta}) * \hat{\boldsymbol{X}}_c(t,\hat{\theta})\right) \quad (14b)$$

$$\mathrm{P}(\hat{\theta}) = \mathrm{P}(\hat{\theta}) - \mathrm{M}(\hat{\theta}) * \boldsymbol{E_c}(\hat{\theta}) * \mathrm{P}(\hat{\theta}) \quad (14c)$$

where $\mathrm{M}(\hat{\theta})$ is the adaptive covariance matrix, P $(\hat{\theta})$ is the adaptive error covariance matrix that is set initially as: $\mathrm{B_c}(\hat{\theta})\ \mathrm{Q}\ \mathrm{B}_c^T(\hat{\theta})$, Q is the scalar process noise covariance and R is the scalar sensor noise covariance, $\boldsymbol{B_C}(\hat{\theta})$ is the adaptive input matrix of the system, $\boldsymbol{E_c}(\hat{\theta})$ is the adaptive output matrix of the system, $\hat{\boldsymbol{X}}_c(t,\hat{\theta})$ is the adaptive state vector and $y(t)$ is the system output signal.

Secondly, the time update period in which the next estimated states and P are calculated depending on the values of (estimated states, $\mathrm{P}(\hat{\theta})$ and $\mathrm{M}(\hat{\theta})$) at current time. The time update equations are presented as follows:

$$\hat{\boldsymbol{X}}_c(t,\hat{\theta}) = \boldsymbol{A_c}(\hat{\theta}) * \hat{\boldsymbol{X}}_c(t,\hat{\theta}) + \boldsymbol{B_C}(t) * u(t,\hat{\theta}) \quad (15)$$

$$\mathrm{P}(\hat{\theta}) = \boldsymbol{A_c}(\hat{\theta}) * \mathrm{P}(\hat{\theta}) * \boldsymbol{A_c}(\hat{\theta}) + \boldsymbol{B_C}(\hat{\theta}) * \mathrm{Q} * \boldsymbol{B_C}(\hat{\theta}) \quad (16)$$

Where:

$\boldsymbol{A_c}(\hat{\theta})$: is the adaptive system matrix.

$u(t,\hat{\theta})$: is the adaptive optimal control signal.

The function of using Adaptive Kalman Filter in this controller is to estimate adaptive system states with filtration to the output signal from the noise that introduced by plant or by measuring sensor equipment.

The algorithm of adaptive linear quadratic Gaussian controller with integral action can be summarized as follows:

Step 1: select Q as a positive definite matrix K_I, and R as an integer value.

Step 2: Read the new values of $y(t)$ and $u(t)$.

Step 3: Estimate the process parameters $\hat{\theta}$ using RLS estimator or RELS estimator and formulate a new identified state – space model of the plant $\{\boldsymbol{A_c}(\hat{\theta}), \boldsymbol{B_c}(\hat{\theta}), \boldsymbol{E_c}(\hat{\theta}), \boldsymbol{C_c}(\hat{\theta})\}$ using equations (8a) and (8b).

Step 4: Calculate Adaptive $P(\hat{\theta})$ matrix using equation (9).

Step 5: Calculate $\boldsymbol{K_c}(\hat{\theta})$ Adaptive gains matrix using equation (10).

Step 6: choose the integral gain $\boldsymbol{K_I}$.

Step 7: Calculate the integral state $X_I(t)$ using equation (12).

Step 8: Compute $\boldsymbol{N}(\hat{\theta})$ using equation (11).

Step 9: Apply the adaptive optimal control signal using equation (13).

Step 10: Estimate Adaptive system states $\hat{\boldsymbol{X}}_c(t,\hat{\theta})$, using equation (15)

Step **1** to **10** are repeated every sampling instant.

V Simulation results:

The objective of this section is to assess the performance and the robustness of the closed loop system using the new adaptive linear quadratic Gaussian controller. The geostationary satellite system model is used to show the ability of the proposed controller, which is enhanced by Kalman filter to track the reference signal in the presence measured load disturbance and set-point change.

The controller parameters Q, KI and R were selected as follows:

$K_1 = 25, Q = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $and\ R = 10.$

A - Investigating the Influence of parameters change on the response of the geostationary satellite:

The following simulation experiment investigates the effect of parameters change on the response of the geostationary satellite, using the developed controller. The dynamics of the process were changed at the 120 sampling instant as illustrated in figures (3),(4)and(5).

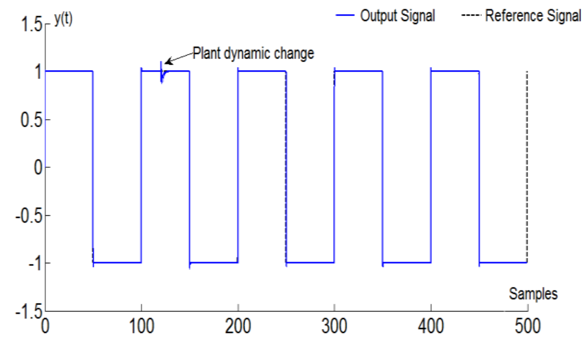

Figure (3) The Geostationary Satellite System Output affected by Parameters Change on the Response

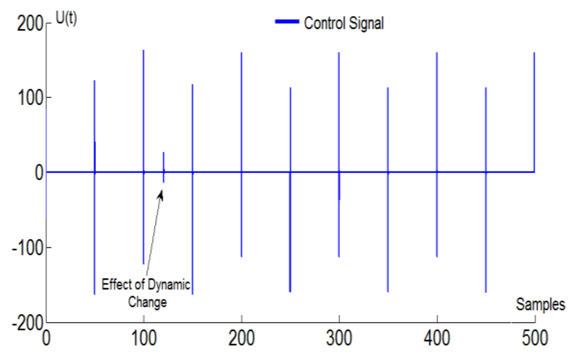

Figure (4) The optimal Control Input of Geostationary Satellite System Output affected by Parameters Change on the Response

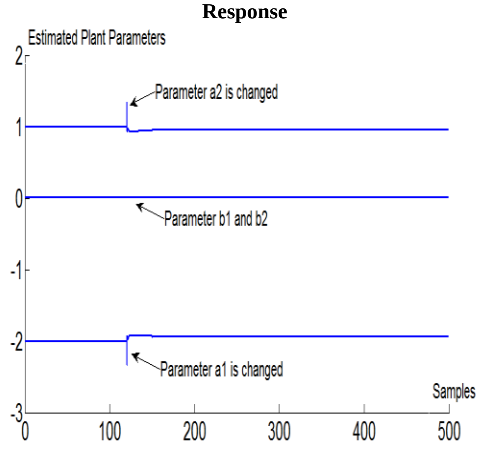

Figure (5) On – Line Geostationary Estimated Satellite System Parameters

It can clearly be seen from figure (3), (4) and (5), that the changing of process dynamic affects the control input and output signals. In addition, the developed controller effectively reposes to the process dynamic changes.

B - Root locus Analysis of the Geostationary Satellite System Response:

The root Locus analysis was presented in figures (6) and (7) presented below.

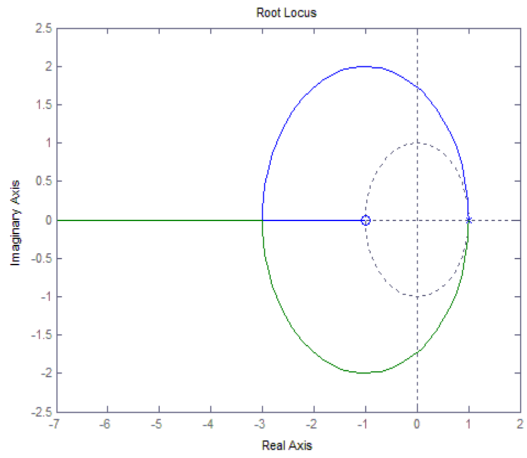

Figure (6) Open Loop Root Locus of the Geostationary Satellite System

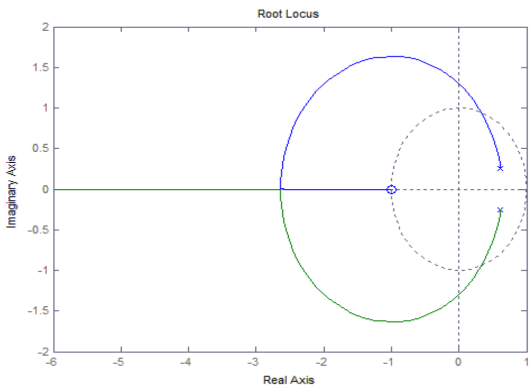

Figure (7) Closed Loop Root Locus of the Geostationary Satellite System with ALQGR controller

It can clearly be seen from figure (6) that, the roots of geostationary satellite open loop system are critical stable, which is an issue in control systems stability. In contrast, figure (7) demonstrates that the poles of open loop system are moved to the optimal location inside the unit circle. This implies that the ALQG enhanced the system stability.

VI Conclusions

In this paper, a new adaptive LQG controller based on discrete state-space technique has been designed. The design was successfully examined on geostationary satellite model. The results presented here indicate that the controller tracks set point changes and at steady-state the controller has the ability to reject constant load disturbances to zero. The design, which is based on state-space representation, retains the simplicity of adaptation mechanism shown on transfer function designs. This advantage allows the controller to be incorporated easily with other transfer function design in a multiple framework. Moreover, the main advantage of the proposed work is that the design which based on new state-space technique can provide more flexibility to deal with non-linear systems.

However, in this paper only single-input single-output case study is considered and the pre-specified control parameters Q, K_I, and R are selected using a trial and error method. A further research may be performed to investigate the possibility of using computational methods such as genetic algorithms or fuzzy logic to obtain the optimal values of these parameters and to extend this work to incorporate multi-input multi-output systems.

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