Design and Simulation Optimal Controller For Quarter Car Active Suspension System

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Abstract–The objectives of this study are to obtain a mathematical model for the active suspension system for Quarter car model. Current automobile suspension systems using passive components only by utilizing spring and damping coefficient with fixed rates. Vehicle suspensions systems typically rated by its ability to provide good road handling and improve passenger comfort. Passive suspensions only offer compromise between these two conflicting criteria. Active suspension system poses the ability to reduce the traditional design as a compromise between handling and comfort by directly controlling the suspensions force actuators. In this thesis, the Proportional Derivative (PD), Linear Quadratic Control (LQR) and Fuzzy logic tune PD controllers' techniques implemented to the active suspensions system for a quarter car model. The Comparison between these controllers by means of the reduction of body displacement trajectory under influence two road profiles and variation of body mass.

Index Terms— **Using classical PD, Linear Quadratic Regulator (LQR) controller, Fuzzy self-tune PD controller.**

I. INTRODUCTION

A suspension system in vehicle is the mechanism that physically separates the vehicle body from the wheels of the vehicle. The performance of the suspension system has been greatly increased due to the increasing of the vehicle capabilities. Ideally the suspension should isolate the body from road disturbances and inertial disturbances associated with cornering and braking or acceleration. The suspension must also be able to minimize the vertical force transmitted to the passengers for their comfort. This objective can be achieved by minimizing the vertical vehicle body acceleration. An excessive wheel travel will result in non-optimum attitude of tire relative to the road that will cause poor handling and adhesion. The main goal of the study is to improve the traditional design trade-off between ride and road handling by directly controlling the suspension forces to suit with the performance characteristics.

The suspension system can be categorized into passive, semi-active and active suspension system according to external power input to the system.

The main objective of this paper, we present a multi-objective control for the active suspension system for quarter car model by using PD, LQR and Fuzzy self-tune PD controller. first method is used to determine the gains of PD and LQR do not guarantee the best performance of the system, which support the important of fuzzy self-tuning method to obtain the parameters of PID for the best Performance. A fuzzy self-tuning is used to develop the optimal control parameters for PID controller to minimize suspension working space of the sprung mass and its change rate to achieve the best comfort of the driver.

II. MATHEMATICALMODEL FORMULATION

1. *Active Suspension System*

This section is devoted to the mathematical modeling of proposed model. Figure 1 shows the two-degrees-of-freedom system that represents the quarter-vehicle active suspension model. It consists of an upper mass M1, representing the body mass (sprung mass), as well as a lower mass M2, representing the wheel mass (un-sprung mass), and its associated parts. The vertical motion of the system is described by the displacements x1, and x2, while the excitation due to road disturbance is w. The suspension spring constant is k1, damping coefficient is b1, and the tyre spring constant is k2 the tyre damping is b2. The data employed here for the quarter-vehicle system are listed in Table 1[1].

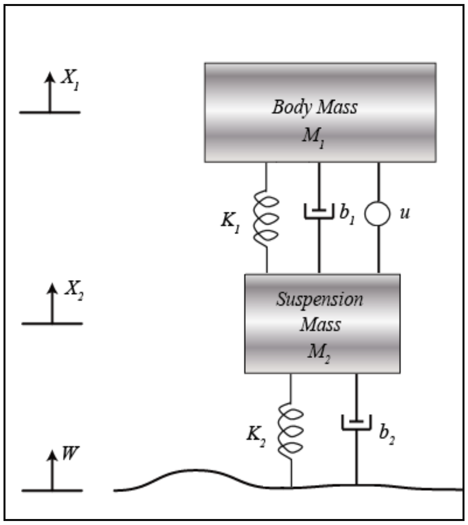

Figure 1. active suspension system.

*T*able 1:parameters of quarter suspension system.

Symbol	Description	Quantity
M_1	1/4 bus body mass	2500 kg
M_2	suspension mass	320 kg
K_1	spring constant of suspension system	80000 N/m
K_2	spring constant of wheel and tire	500000 N/m
b_1	damping constant of suspension system	350 N.s/m
b_2	damping constant of wheel and tire	15020 N.s/m
u	control force	N

By applying Newton's second law the equations of motion for the sprung and unsprung mass of the quarter-car suspension model are given by [2]:

$$M_1\ddot{X}_1 = -b_1(\dot{X}_1 - \dot{X}_2) - k_1(X_1 - X_2) + U \qquad (1)$$

$$M_2\ddot{X}_2 = -b_1(\dot{X}_1 - \dot{X}_2) + K_1(X_1 - X_2) + b_2(\dot{W} - \dot{X}_2) + K_2(W - X_2) - U \qquad (2)$$

Where:

$M1 = 1/4\ bus\ body\ mass$

$M2 = suspension\ mass$

$K1 = spring\ constant\ of\ suspension\ system$

$K2 = spring\ constant\ of\ wheel\ and\ tire$

$b1 = damping\ constant\ of\ suspension\ system$

$b2 = damping\ constant\ of\ wheel\ and\ tire$

$U = control\ force$

$W = road\ disturbance/road\ profile$

$X1 = car\ body\ displacement$

$X2 = wheel\ displacement$

Since the distance X_1-W is very difficult to measure, and the deformation of the tire (X_2-W) is negligible, we will use the distance X_1-X_2instead of X_1-W as the output in our problem. Keep in mind that this is estimation [1].
From the main the dynamic equations the state-space as the following [2].

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{y}_1 \\ \ddot{y}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-b_1 b_2}{M_1 M_2} & 0 & \left[\frac{b_1}{M_1}\left(\frac{b_1}{M_1} + \frac{b_1}{M_2} + \frac{b_2}{M_2}\right)\right] & \frac{-b_1}{M_1} \\ \frac{b_2}{M_2} & 0 & -\left(\frac{b_1}{M_1} + \frac{b_1}{M_2} + \frac{b_2}{M_2}\right) & 1 \\ \frac{K_2}{M_2} & 0 & -\left(\frac{K_1}{M_1} + \frac{K_1}{M_2} + \frac{K_2}{M_2}\right) & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ y_1 \\ \dot{y}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M_1} \\ 0 \\ \left(\frac{1}{M_1} + \frac{1}{M_2}\right) \end{bmatrix}$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ y_1 \\ \dot{y}_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} U \\ W \end{bmatrix} \qquad (4)$$

For the active suspension, the actuator force u depends on the time histories of the relative displacement y_1 across it:

$$y_1 = x_1 - x_2 \qquad (5)$$

Where, y is also referred to as the suspension travel. In this paper, it is assumed that only the suspension working space could be measured and used by the controllers
Figure 2 illustrates the MATLAB/SIMULINK model of active suspension system.

2. *Road Excitation Profile*

Road profile is considered by unit step with amplitude, chosen to measure step response.

III. Controllers

1. *Classical Proportional Derivative(PD) Controller*

Trial and error method to tune the coefficients of a PD controller became the focus of research and become better understood, but cannot guarantee to be always effective. For this reason, this paper investigates the design of self-tuning for a PD controller. The transfers function of the PD controller [3]given as

$$u_{PID}(s) = E(s)(k_P + k_d s) \qquad (6)$$

Where, k_P is proportional gain, K_d is the derivative gain. Figure 3 illustrates the MATLAB/SIMULINK model of active suspension system with PD controller.

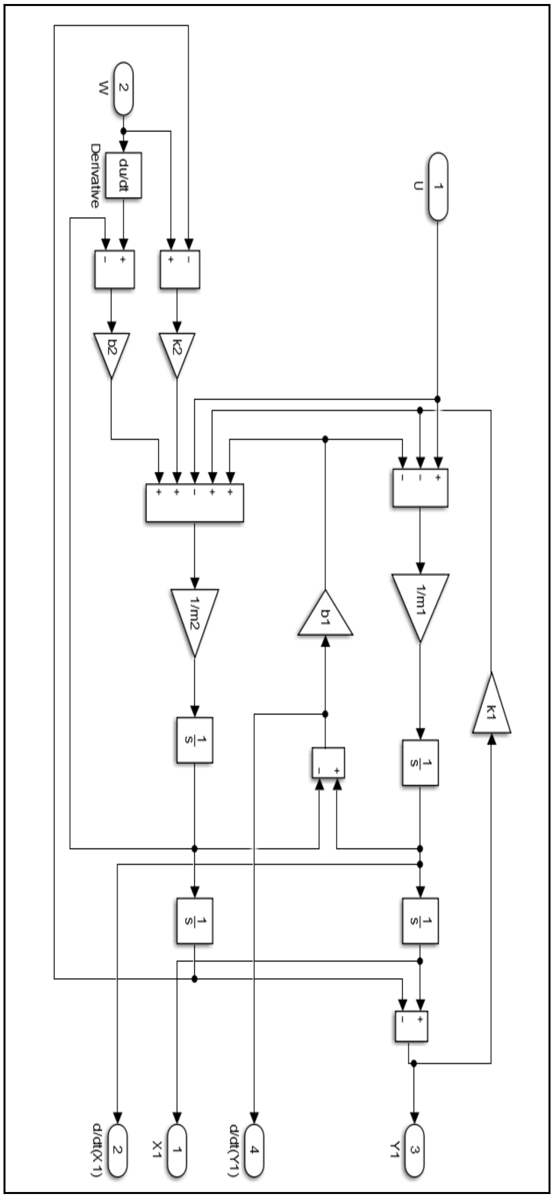

Figure2. Quarter active suspension system using SIMULINK.

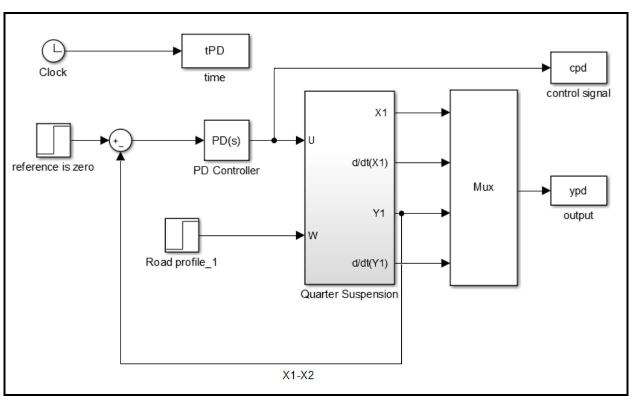

Figure3.active suspension system with PD controller.

PD controller, a kind of linear controller, which include two controlled variables, proportion (kP) and derivative (kd) through linear combination. The value of PD gains must be tuned at certain values of model parameters.

The design was based on trial and error method the tuning of the controller.

$$\mathbf{kP = 1000 \qquad kd = 33000}$$

The step response of close loop system with PD controller is shown in figure 4. The percent overshoot is 28.82 %, the steady state error is zero. Where settling, time is 0.705 sec.

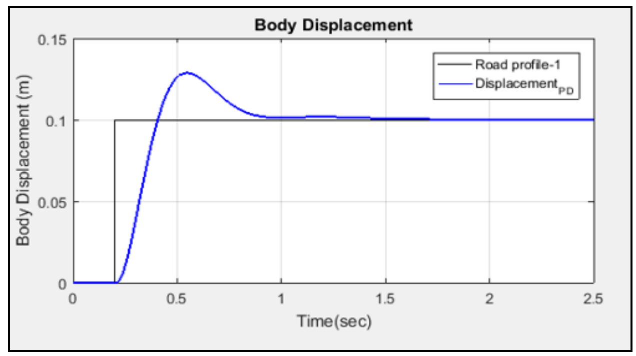

Figure 4. body displacement with PD controller.

2. *Linear Quadratic Regulator (LQR) controller*

LQR approach is helpful in weighing factor of performance index in accordance with designer's desires and constraints. For state space LQR controller design, state variables of active suspension system are re-presented in equation 3 and 4.

Figure 5 illustrates the MATLAB/SIMULINK model of active suspension system with state variable feedback.

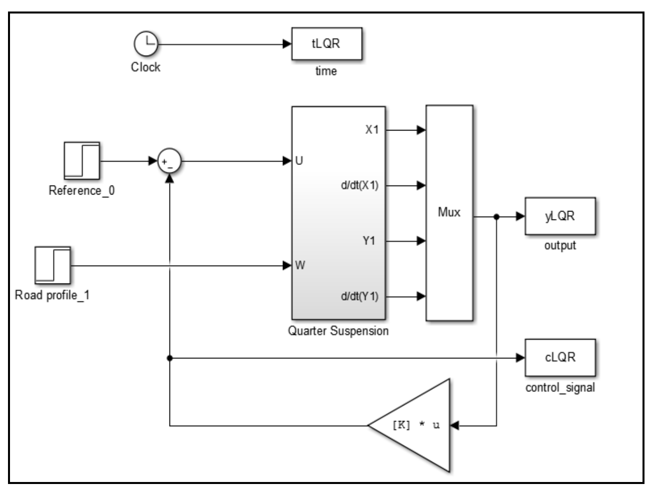

Figure 5.active suspension system with LQR controller.

Linear feedback of controller is governed through equation 7.

$$\mathbf{u} = -\mathbf{K}\mathbf{x} \qquad (7)$$

State feedback gain is denoted by K [4].
MATLAB command used for obtaining suitable LQR controller,

Matrices (Q) and (R) were tuned using trial and error until simulation results were displaying desired system performance.

$$Q = \begin{bmatrix} 1\times10^5 & 0 & 0 & 0 \\ 0 & 2\times10^6 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 1000 \end{bmatrix}$$

$$R = 0.005$$

The LQR controller gives a much stable and robust response more than PD for the system. The response of the system with LQR controller is shown in figure 6.

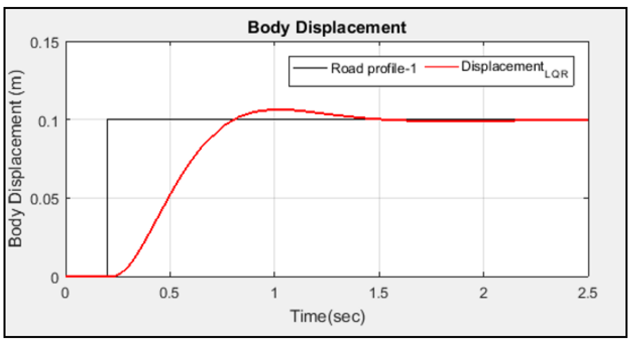

Figure6. body displacement with LQR controller.

3. *Fuzzy Logic Self-Tune (PD) Controller*

The fuzzy used to tune PD controller to control suspension system as shown in figure 4.3 has two inputs; the error ($e = x_1 - x_2$) suspension travel, and change in error ($\dot{e} = \frac{de}{dx}$) and two outputs; proportional and derivative controller gains.

Thefuzzy controller consists of four elements: fuzzification interface. Role base, inference mechanism and defuzzification interface.

Therefor in order to design a fuzzy controller, these elements must be designed:

a. *Fuzzification design:*

The membership functions chosen for two crisp inputs are triangular membership functions used to carry out the fuzzification process for both e and e˙ which are shown respectively in figure 7. and figure 8, where in each fuzzy sets the corresponding designed universe of course range for considered suspensions system is specified.

The fuzzy set of each input variable is represented by five linguistic variables which as (NB, NM, Z, PM and PB) for (Negative Big, Negative Medium, Zero, Positive Medium and Positive Big) respectively. And fuzzy sets output membership function of both output of the fuzzy controller also used triangular membership functions represented by three linguistic variables which as (S, M and B) for (Small, Medium and Big) shown in figure 9.

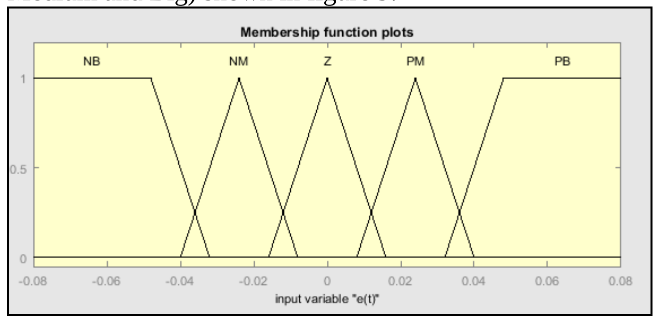

Figure7. Error membership function.

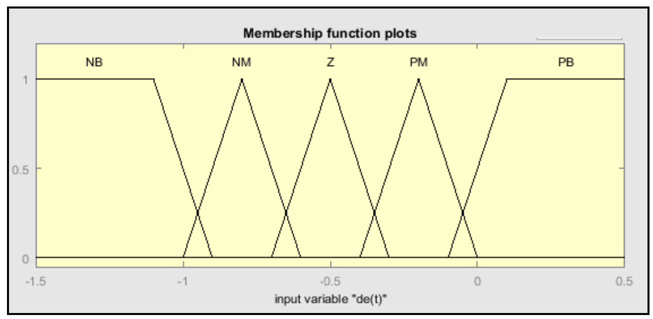

Figure8.Change of Error membership function.

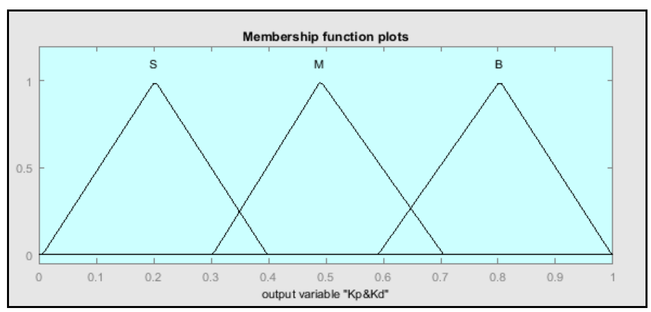

Figure9.Proportional (Kp) gain and derivative (Kd) membership functions.

b. *Rule base design:*

The rule base used in tuning PD is shown in table 2. and table 3. where five fuzzy sets variables are chosen for two-dimensional rule table results from the two inputs e and e˙ as result 25 rules well be obtained (5^2) which are designed from the expert knowledge and they are typically expressed as follows.

$$\text{if } e \text{ is } N_{e_j} \text{ and } \dot{e} \text{ is } N_{\dot{e}_j} \text{ then } u_a \text{ is } u_{f_j}$$

Where u_{f_j} is the j^{th} fuzzy output center rule, N_{e_j} and $N_{\dot{e}_j}$ represent respectively the linguistic variables for e and $\dot{e}$. The logic operation in the designed fuzzy rules that combines the meaning of two linguistic inputs can be represented by minimum operation. This type of process is known as inference mechanism [5].

Table 2.Rule base for Kp.

e(t) / de(t)	NB	NM	Z	PM	PB
NB	B	M	S	M	B
NM	B	M	S	M	B
Z	B	M	S	M	B
PM	B	M	S	M	B
PB	B	M	S	M	B

Table 3.Rule base for kd.

e(t) / de(t)	NB	NM	Z	PM	PB
NB	B	B	B	B	B
NM	M	M	M	M	M
Z	S	S	S	S	S
PM	M	M	M	M	M
PB	B	B	B	B	B

c. Defuzzification design:

The last part of fuzzy controller is the defuzzification operation which evaluates the crisp outputs value via the center of gravity method, which simply can be described by the following formula[6]:

$$u_a = \sum_{j=1}^{n}\left(\frac{\mu_j u_{f_j}}{\mu_j}\right) \qquad (8)$$

Where: n is the number of rules, μ_j is the output of minimum operation, and u_{f_j} is the rule center which represents the conclusion of the j^{th} rule.

Self-tuning fuzzy PD regulator subsystem block as shown in Figure 10 consists of Fuzzy and PD block with some modification refers to the formula which is applied to calibrate the value of K_p and K'_d from fuzzy block to obtain the value of K_p, and K_d. Each parameter has its own calibration [7]. While, the complete Simulink block for whole system including the control design and the plant is shown in Figure 11

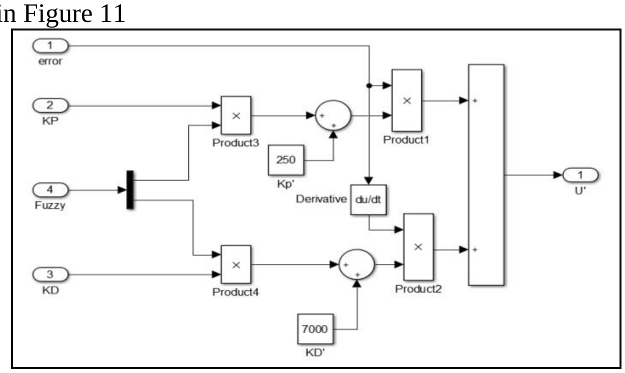

*Figure 10.*PD with external parameters.

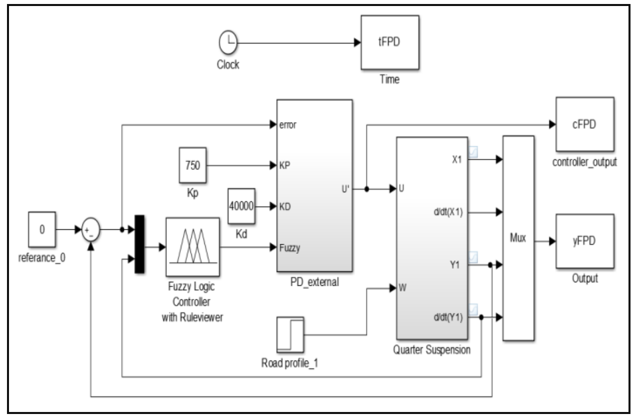

Figure 11.active suspension system with Fuzzy self-tune PD controller.

The value of parameter Kp and Kd are tuned by using signals from fuzzy logic block based on the error and the changes of error between reference signals and output signals. In order to perform the output of the system, a step inputsignal is applied.

The outputs of the simulation for step input are represented in Figure 12.

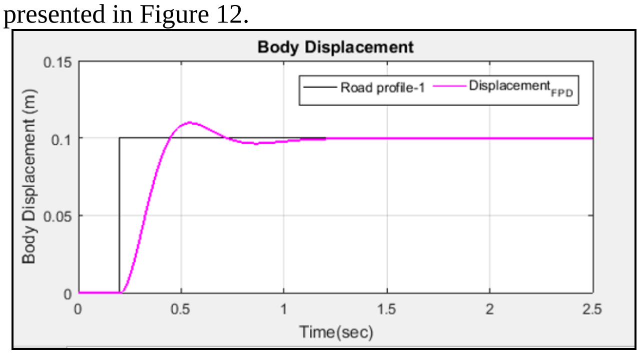

Figure 12.body displacement with Fuzzy self-tune PD controller.

Figure 12. shows the performance of the control system with respect to step reference input signal. The self-tuning fuzzy PD controller achieves better response. It is indicated from fast rise time, faster settling time, less overshoot and without steady state error.

IV. Comparison of simulation results

Comparison between the performance of the PD, LQR and Fuzzy tune PD controllers are evaluated in terms of body displacement, body displacement velocity, where in all subsequent figures the solid blue line represents PD, solid red line represents LQR controller and solid green line represents Fuzzy tune PD controller.

In this section comparison of all the three controllers based on simulation Results are discussed. Parameters compared are percentage peak over-shoot, rise and settling time.

Where the body displacement, displacement-velocity and the suspension travel obtain in following Figures 13, Figure 14 and figure 15.

From the simulation results, the body displacement and body displacement velocity performances of Fuzzy PD compared to classical PD and LQR controller are shown in Figures 13 and figure 14.it is clear that the active system with Fuzzy PD is able to significantly reduce both amplitude and the settling time of unwanted body motions in the forms of body displacement and body displacement velocity as compared with the counterpart.

Table 4.Comparison in time response of body displacement.

	PD	LQR	FPD
RiseTime(sec)	0.136	0.384	0.171
SettlingTime(sec)	0.710	1.183	0.588
Overshoot(%)	28.80	6.80	11.09
Peak(m)	0.128	0.1063	0.111
PeakTime(sec)	0.344	0.784	0.3751

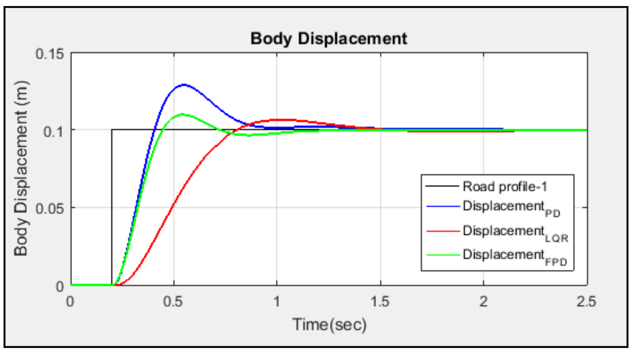

Figure 13.Body-displacement.

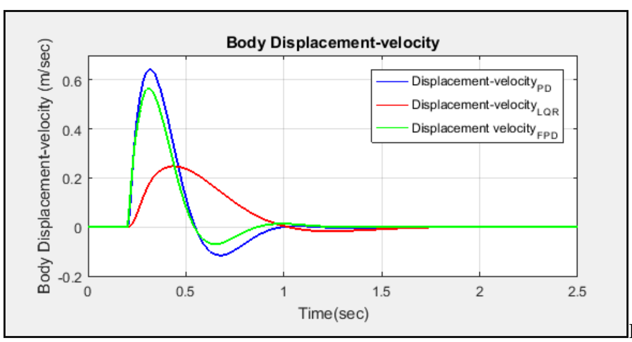

Figure 14. body displacement-velocity.

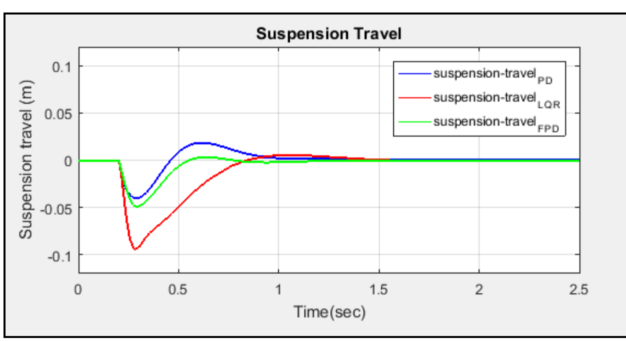

*Figure 15.*Suspension travel.

The suspension deflection (suspension travel) performance as shown in Figure 15, in which the active system with each controller shows significant performances in reducing both amplitude and the settling time compared with the passive system.
The output of the controller of Figure 5.18 it is noted that values of actuator force for Fuzzy PD is slightly lower than classical PD while in LQR it is low compare to others.

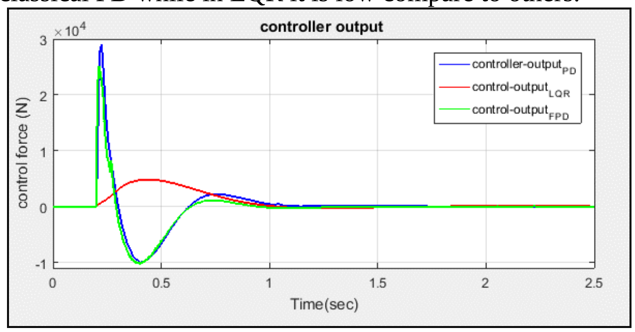

Figure 16.controller output.

V. *Conclusion*

The objectives of this project have been achieved. Dynamic model for linear Quarter car active suspensions systems has been formulated. The Proportional Derivative (PD), Linear Quadratic Regulator (LQR) and Fuzzy Logic Tune PD controller are presented and used to reduce the effects of road disturbances on the linear quarter car model.

The performance characteristics and the robustness of the active suspension system are evaluated and then compared between these controllers.

The results show that the use of the three proposed control techniques are effective in controlling a vehicle but Fuzzy logic tune PD controller more robust compared to the two other controllers. From the simulation results, it can be seen that the proposed controller shows improvement in reducing both magnitude and settling time of the body displacement, body displacement velocity and suspension travel. The proposed controller is capable of satisfying all the requirements for active suspension design.

VI. *References*

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